Experimental investigation of the mass flow gain factor in a draft tube with cavitation vortex rope

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Abstract. At off-design operating operations, cavitating flow is often observed in hydraulic machines. The presence of a cavitation vortex rope may induce draft tube surge and electrical power swings at part load and full load operations. The stability analysis of these operating conditions requires a numerical pipe model taking into account the complexity of the two-phase flow. Among the hydroacoustic parameters describing the cavitating draft tube flow in the numerical model, the mass flow gain factor, representing the mass excitation source expressed as the rate of change of the cavitation volume as a function of the discharge, remains difficult to model. This paper presents a quasi-static method to estimate the mass flow gain factor in the draft tube for a given cavitation vortex rope volume in the case of a reduced scale physical model of a $\nu = 0.27$ Francis turbine. The methodology is based on an experimental identification of the natural frequency of the test rig hydraulic system for different Thoma numbers. With the identification of the natural frequency, it is possible to model the wave speed, the cavitation compliance and the volume of the cavitation vortex rope. By applying this new methodology for different discharge values, it becomes possible to identify the mass flow gain factor and improve the accuracy of the system stability analysis.

1. Introduction

The integration of the renewable energy resources has a significant impact on the stability of the electrical grid. The fast changes in power generation by renewable energy resources require a flexible operating range of the hydro units. At off-design operating conditions of hydraulic machines, the onset of a cavitation volume attached to the runner hub is often observed. At part load condition, a draft tube surge and electrical power swings [7], [5] occurs when the vortex rope frequency matches the eigenfrequencies of the hydraulic system. At full load operation, self-oscillations are experienced as an axial pulsation of the cavitation volume corresponding to the one of the eigenfrequency of the system and may cause negative damping [9], [14]. The behavior of full load surge in model test is reported by Prenat and Jacob in 1986 [16]. Koutnik et al. [10] successfully simulate full load surge using a simplified 1D mathematical model of the full-load vortex rope and using the cavitation compliance and the mass flow gain factor (MFGF) parameters [4]. More recently, by deriving the compliance and the MFGF parameters from CFD simulations of the complete machine, Flemming et al. [6] predicted instability of a prototype installation based on eigenvalues and eigenmodes analysis.

In this paper, a quasi-static method estimates the MFGF using experimental data and a one-dimensional draft tube model developed by Alligné et al. [1]. On the one hand, the methodology is based on an experimental identification of the natural frequency of a ν = 0.27 Francis turbine for different Thoma numbers and speed factor n_{ED}. On the other hand, the numerical model of the draft tube is derived from momentum and continuity equations including the convective terms and the divergent geometry of the draft tube to reproduce the dynamic behavior of the two-phase flow. This model requires knowledge of the wave speed, the bulk viscosity induced by the cavitation vortex rope and the MFGF. Landry [12] [13] developed a methodology and dimensionless power laws to model the wave speed and the bulk viscosity for a given Francis turbine at part load condition.

2. One dimensional modeling of the cavitation vortex rope dynamics

A one dimensional numerical model to simulate the behavior of the cavitation vortex rope is used, taking into account the dissipation induced by the cavitation vortex rope, the divergent geometry of the draft tube and the convective term of the momentum equation. The test rig hydroacoustic model for the numerical simulations is made with the SIMSEN software developed by EPFL. The modeling of the hydraulic components is based on an equivalent electrical scheme representation, see
Figure **1**.

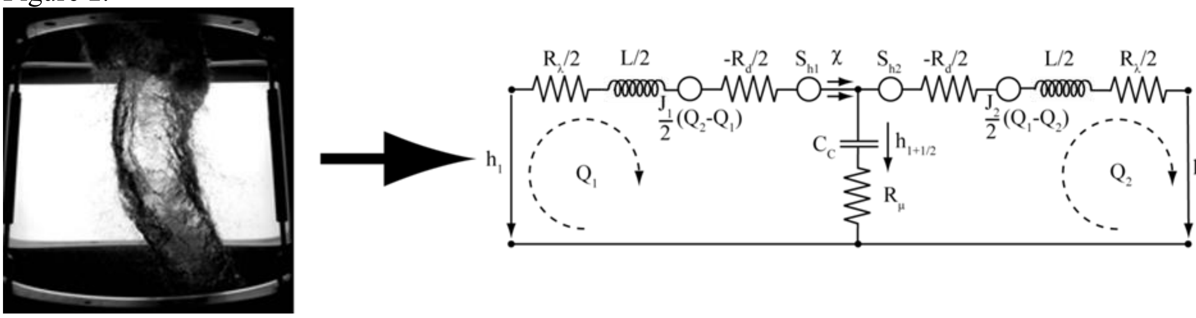


Figure 1. Visualization of the cavitation vortex rope in the draft tube cone and the corresponding equivalent circuit.

The momentum and the continuity equations are solved using the Finite Difference Method with 1st order centered scheme discretization in space and a scheme of Lax for the discharge state variable. This discretization yields a system of ordinary differential equations that can be represented as a T-shaped equivalent circuit. For the hydroacoustic model of the cavitation vortex rope, uniform pressure and velocity distributions in a cross section A are assumed. The parameters of the equivalent circuit are widely described by Alligné et al. [1]:

- The mathematical development of the cavitation volume fluctuations yields a dynamic parameter called cavitation compliance C_c. This dynamic parameter represents the variation of the cavitation volume V_c with respect to a variation of pressure and implicitly defines the local wave speed a in the draft tube, influencing the traveling time of pressure waves.

$$C_c = -\frac{\partial V_c}{\partial h_{i+\frac{1}{2}}} = \frac{gAdx}{a^2} \tag{1}$$

- The MFGF represents the excitation mass source of the hydraulic system and is defined as the cavitation volume variation as a function of a discharge variation Q_1 at the runner outlet.

$$\chi = -\frac{\partial V_c}{\partial Q_1} \tag{2}$$

The flow mechanism determining the MFGF is not clear and is therefore difficult to model.

3. Methodology

The methodology is based on the direct link between the natural frequency of the hydraulic system and the local wave speed in the draft tube [8], [12]. The first step is to identify the natural frequency of the hydraulic system for a given operating point. To experimentally identify the natural frequency, an excitation system was designed to inject or extract a periodical hydroacoustic power at a given frequency. This excitation system is an upgraded version of the rotating valve used by Blommaert [3] to actively control the hydraulic fluctuations of a Francis turbine at part load operating conditions. The excitation system and the spectral analysis were widely described in [12]. With pressure sensors located on the hydraulic system, the natural frequency can be identified with a spectral analysis of the forced harmonic response. Then, the wave speed value can be adjusted in the draft tube modeling to obtain a numerical natural frequency similar to the experimental natural frequency. Numerically, the natural frequency is defined by conducting an eigenvalue study of the non-linear hydraulic system. The eigenvalues are based on the linearization of the set of equations around the operating point [2]. Damping and oscillation frequency of the eigenmodes are respectively given by the real part and the imaginary part of the eigenvalues.

After the identification of the wave speed, the second step is to compute the value of the cavitation vortex rope V_c, defined as the volume limited by the vapor pressure iso-surface. With equation (1), the cavitation compliance can be rewritten as a function of the Thoma number [11]:

$$C_C = -\frac{\partial V_C}{\partial h_{\bar{1}+\frac{1}{2}}} \approx -\frac{\partial V_C}{\partial h_{\bar{I}}} = -g\frac{\partial V_C}{\partial NPSE} = -\frac{\partial V_C}{\partial NPSH} = -\frac{\partial V_C}{\partial\left(\sigma \cdot H_{Turb}\right)} \Rightarrow -\frac{\partial V_C}{\partial\left(\sigma \cdot H_{Turb}\right)} = \frac{gAdx}{a^2} \qquad (3)$$

Moreover, for a given operating point, *i.e.* Q_{ED} and n_{ED} are constant, the cavitation volume can be computed by integrating the equation (3). With this information, it becomes possible to compute the cavitation volume and the void fraction β, defined as the fraction of a reference volume V_{ref} that is occupied by the gas phase volume V_c. The reference volume is defined by the numerical elements where the cavitation compliance is adjusted and corresponds to the cone and the first half of the elbow volumes of the draft tube modeling. The void fraction values are finally validated by analyzing the visualization of the cavitation vortex rope with a high-speed camera.

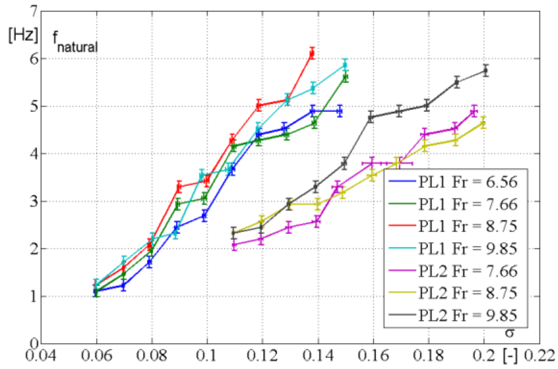
The last step to determine the value of the MFGF needs the identification of cavitation volume for different discharge values. Thus, for a constant guide vane opening and discharge factor, the discharge value is modified by increasing the rotational speed of the hydraulic turbine. By identifying the cavitation volume for 3 different speed factors, the MFGF can be computed with equation (2) by using the Finite Difference Method.

4. Results and discussion

For a fixed set of values of the discharge factor Q_{ED}, the Froude and the Thoma numbers, see Table 1, the hydraulic system is experimentally excited in a frequency range around the natural frequency, in order to determine the eigenmode of the test rig. For each excitation frequency, the forced harmonic response of the hydraulic system is measured with the 27 pressure sensors located throughout the test rig. Figure 2 (Left) shows the relation between the Thoma number and the first eigenfrequency of the hydraulic system obtained by applying the same methodology for all Thoma numbers, Froude numbers and Q_{ED} values for operating conditions *PL1* and *PL2*. Globally, the natural frequency increases with the Thoma number when the volume of the cavitation vortex rope decreases. In addition to that, the Froude number influence remains small and its decrease causes a slight reduction of the eigenfrequency.

Table 1. Selected Francis turbine operating conditions

		$n_{ED}/n_{ED,BEP}$	$Q_{ED}/Q_{ED,BEP}$	*Fr*	σ
FL 1	Full Load	[0.91, 0.95, 1]	1.3	5.79	[0.12 – 0.22]
PL 1	Part Load	[0.95, 1, 1.04]	0.80	[6.56, 7.66, 8.75, 9.85]	[0.06 - 0.15]
PL 2	Part Load	[0.95, 1, 1.04]	0.64	[7.66, 8.75, 9.85]	[0.10 - 0.20]
PL 3	Part Load	[0.95, 1, 1.04]	0.53	7.66	[0.08 - 0.14]
PL 4	Part Load	[0.95, 1, 1.04]	0.725	6.56	[0.06 - 0.13]



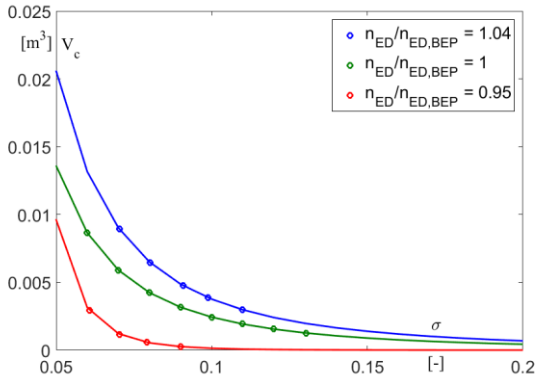


Figure 2. Influence of the Thoma number on the natural frequency for the operating point PL1 and PL2 (Left) and evolution of the cavitation volume for the operating point PL1 (Right).

By analyzing the evolution of the cavitation compliance as a function of the Thomas number, it was observed that the values of the cavitation compliance can be interpolated with a power regression [11]. By integrating this power regression, the cavitation volume and therefore the void fraction can be directly computed. The void fraction obtained is compared with the void fraction estimated with a high-speed visualization of the cavitation vortex rope in the Plexiglas cone. The experimental method based on the image processing of a high-speed flow visualization was already used by Müller et al. [15] to determine the cavitation volume oscillation on a microturbine featuring a self-oscillating vortex rope in its conical draft tube. The experimental estimation of the vortex rope volume is based on the detection of the edges of the cavity. The accuracy of this analysis relies on proper lighting conditions and, therefore, accuracy depends on sharp contrast between the liquid and the gaseous phase of the flow. The numerical value of the void fraction is in good agreement with the experimental values deduced from the high-speed camera, see Figure 3 (Left).

After validating the cavitation volume with experimental visualization, it is now possible to compute the MFGF with equation (2). For full load conditions, the MFGF is negative, see Figure 3 (Right). Indeed, by increasing the discharge, the tangential velocity in the draft tube cone increases. Therefore, the pressure in the vortex core decreases and the cavitation volume increase. On the contrary, at part load conditions, the MFGF is positive. The tangential velocity in the draft tube cone decreases when the discharge increases. Thus, the pressure in the vortex core increases and the cavitation volume decreases. At deep part load, the cavitation volume increases with the discharge and the MFGF behaves similarly to full load conditions. Finally, these properties of the MFGF are amplified when the Thoma number decreases.

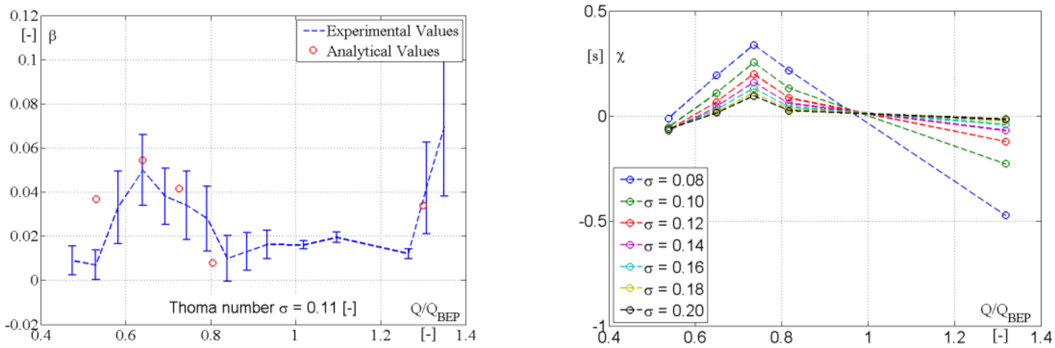


Figure 3. Comparison between the numerical and the experimental values of the void fraction for different discharge factor (Left) and evolution of the MFGF as function of the discharge value for different Thoma numbers (Right).

5. Conclusion

This paper presents a quasi-static method to estimate the MFGF in the draft tube for a given cavitation vortex rope volume in the case of a reduced scale model of a Francis turbine. The methodology is based on an experimental identification of the natural frequency of the hydraulic system for different Thoma numbers and speed factors n_{ED}. With the identification of the natural frequency, it is possible to model the cavitation compliance and the volume of the cavitation vortex rope. This volume was confirmed by analyzing the visualization of the cavitation vortex rope with a high-speed camera. By applying this methodology for different speed factors, it becomes possible to identify the MFGF. Finally, the next step is to define a dimensionless number in order to generalize the MFGF and predict its value for all operating points. This will ultimately pave the way to a more precise stability analysis of hydropower plants.

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